

Internal Charmonium Evolution in the Quark-Gluon Plasma

Baoyi Chen¹, Xiaojian Du², Ralf Rapp²

¹*Department of Physics, Tianjin University, Tianjin 300350, China*

²*Cyclotron Institute and Department of Physics and Astronomy,
Texas A&M University, College Station, TX 77843-3366, USA*

We employ a time-dependent Schrödinger equation to study the evolution of a $c\bar{c}$ dipole in a quark-gluon plasma (QGP). Medium effects on the heavy-quark potential in the QGP are found to significantly affect the timescales of the internal evolution of the dipole. Color-screening can enhance the overlap of the expanding wavepackage with excited states at high temperature, while it is reduced at lower temperatures where the dipole favors the formation of the charmonium ground state. We investigate the consequences of this mechanism on the double ratio of charmonium nuclear modification factors, $R_{AA}^{\psi(2S)}/R_{AA}^{J/\psi}$, in heavy-ion collisions. The impact of the transition mechanisms on this ratio turns out to be rather sensitive to the attractive strength of the potential, and to its temperature dependence.

PACS numbers: 25.75.-q, 12.38.Mh, 24.85.+p

I. INTRODUCTION

The anomalous suppression of J/ψ production in relativistic heavy-ion collisions has been considered as a signal of the existence of deconfined matter, i.e., the “Quark-Gluon Plasma” (QGP) [1]. In nucleus-nucleus collisions, the production of charmonia is affected by interactions within the incoming nuclei (cold nuclear matter, CNM) and the subsequently formed medium (hot nuclear matter, HNM). The former include a shadowing of parton distribution functions, Cronin effect and nuclear absorption. They occur prior to the formation of QGP and, at high energies, usually do not involve charmonium eigenstates, but rather pre-resonance states which therefore result in a similar modification of the eventually formed eigenstates. The HNM effects include inelastic collisions with partons, color screening and the recombination of charm and anti-charm quarks in QGP. Differences between the measured nuclear modification factors (R_{AA}) of J/ψ and $\psi(2S)$ should mainly be caused by the hot medium. The sequential suppression idea, based on the hierarchy of binding energies, suggests that $\psi(2S)$ should suffer much stronger suppression than J/ψ in QGP, due to its smaller binding energy [2], with estimated dissociation temperatures of $T_d^{J/\psi} \sim 1.5\text{--}2T_c$ and $T_d^{\psi(2S)} \sim T_c$ [2]. This would mean that the $\psi(2S)$ can hardly survive in QGP and produce a small $R_{AA}^{\psi(2S)}$. Indeed, applying an instantaneous melting picture with the above quoted temperature, transport model calculations result in a double ratio of $R_{AA}^{\psi(2S)}/R_{AA}^{J/\psi} \sim 0.1$ in central Pb-Pb ($\sqrt{s_{NN}}=2.76$ TeV) collisions for $6.5\text{ GeV}/c < p_T < 30\text{ GeV}/c$ [3]. This underestimates the experimental data significantly [4]. One idea to improve on this situation is to include regeneration processes for the ψ' , which, because of its smaller dissociation temperature, would likely be operative near the hadronization transition and in the subsequent hadronic

phase [5]. On the other hand, both QGP and the initially formed $Q\bar{Q}$ pairs in heavy-ion collisions have time evolution scales which can interfere and cause significant deviations from an equilibrium evolution [6, 7]. This situation is reinforced if the binding potential is weakened due to color screening, and for high-momentum charmonia due to time dilation effects [8–10]. Instead of being an eigenstate, the $c\bar{c}$ dipole evolves as a superposition of eigenstates through the medium. Most currently used quarkonium transport models [10–15] do not account for transitions between ground and excited states. While this is probably a good approximation for the predominantly produced ground states, a “dynamical feed-down” may have significant effects on the yields of excited states [16]. The composition will not only change with time but also as a consequence of medium effects on the potential, which specifically may enhance the overlap with the larger-size excited states.

II. TIME DEPENDENT SCHRÖDINGER EQUATION

Since the internal motion of c and \bar{c} inside charmonium bound states is approximately non-relativistic, we employ the time-dependent Schrödinger equation to describe its wavefunction evolution. We explore the in-medium heavy-quark (HQ) potential, $V(r, t)$, within two limits, the free ($F(r, T)$) and internal $U(r, T)$ energies, for which we can employ results from lattice QCD [17]. After separating off the angular parts, the radial time-dependent Schrödinger equation for the quarkonium wavefunction can be written as

$$i\hbar \frac{\partial}{\partial t} \psi(r, t) = \left[-\frac{\hbar^2}{2m_\mu} \frac{\partial^2}{\partial r^2} + V(r, t) \right] \psi(r, t) \quad (1)$$

where r is the relative distance between c and \bar{c} , t is the proper time in the center of mass frame, and $m_\mu = m_1 m_2 / (m_1 + m_2) = m_c/2$ the reduced mass.

To numerically solve Eq. (1), we employ the Crank-Nicholson method [18], which utilizes a discretization of the time evolution as $\mathbf{T}^{n+1}\psi^{n+1} = \Gamma^n$ with

$$\begin{pmatrix} \mathbf{T}_{0,0}^{n+1} & \mathbf{T}_{0,1}^{n+1} & 0 & 0 & \dots \\ \mathbf{T}_{1,0}^{n+1} & \mathbf{T}_{1,1}^{n+1} & \mathbf{T}_{1,2}^{n+1} & 0 & \dots \\ 0 & \mathbf{T}_{2,1}^{n+1} & \mathbf{T}_{2,2}^{n+1} & \mathbf{T}_{2,3}^{n+1} & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix} \begin{pmatrix} \psi_0^{n+1} \\ \psi_1^{n+1} \\ \psi_2^{n+1} \\ \psi_3^{n+1} \\ \dots \end{pmatrix} = \begin{pmatrix} \Gamma_0^n \\ \Gamma_1^n \\ \Gamma_2^n \\ \Gamma_3^n \\ \dots \end{pmatrix}. \quad (2)$$

The non-zero entries are given by

$$\begin{aligned} \mathbf{T}_{j,j}^{n+1} &= 2 + 2a + bV_j^{n+1} \\ \mathbf{T}_{j,j+1}^{n+1} &= \mathbf{T}_{j+1,j}^{n+1} = -a \\ \Gamma_j^n &= a\psi_{j-1}^n + (2 - 2a - bV_j^n)\psi_j^n + a\psi_{j+1}^n \end{aligned} \quad (3)$$

with $a = i\Delta t/(2m_\mu(\Delta r)^2)$ and $b = i\Delta t$.

From the $c\bar{c}$ dipole wavefunction, $\psi(r, t) = rR(r, t)$, one can obtain the coefficient, $c_{mS}(t)$ ($m=0,1,\dots$), of a S -wave charmonium eigenstate in the $c\bar{c}$ dipole at each time step as

$$c_{mS}(t) = \int R_{mS}(r) e^{-iE_{mS}t} \psi(r, t) \cdot r dr. \quad (4)$$

The pertinent nuclear modification factor of an eigenstate evolving in a QGP fireball can be defined as

$$R_{AA}^{mS}(t) = \frac{\int d\vec{x} d\vec{p} |c_{mS}(t, \vec{x}, \vec{p})|^2 \frac{dN_{c\bar{c}}}{d\vec{x}_0 d\vec{p}_0} \mathcal{S}(\vec{x}_0, \vec{p}_0)}{\int d\vec{x}_0 d\vec{p}_0 |c_{mS}(0, \vec{x}_0, \vec{p}_0)|^2 \frac{dN_{c\bar{c}}}{d\vec{x}_0 d\vec{p}_0}} \quad (5)$$

where \vec{x}_0 and \vec{p}_0 are the initial position and momentum of the center of $c\bar{c}$ dipole, with $\vec{x} = \vec{x}_0 + \vec{v}_{c\bar{c}}t$, $\vec{v}_{c\bar{c}} = \vec{p}_0/E_{c\bar{c}}$ and $\vec{p} = \vec{p}_0$ since we neglect in/elastic interactions of the $c\bar{c}$ dipole with the medium. For the initial $c\bar{c}$ phase space distribution, $dN_{c\bar{c}}/d\vec{x}_0 d\vec{p}_0$, one usually assumes a nuclear collision profile, includes a shadowing factor, and samples it using Monte-Carlo techniques. However, we here constrain ourselves to evolve a $c\bar{c}$ dipole in a static medium, to focus on the color screening effects on the internal evolution of the wavepackage. The direct nuclear modification factor then simply becomes $R_{AA}^{mS}(t) = |c_{mS}(t)|^2/|c_{mS}(0)|^2$.

Finally, we need to specify our input for the initial $c\bar{c}$ dipole wavefunction. It can be a sharp Gaussian function during the production time, $\tau_{c\bar{c}} \lesssim 0.1$ fm/c. However, in pp collisions, one should recover the observed fractions of charmonium states. This can be achieved by making a Gaussian ansatz,

$$\phi_0(r) = A \exp\left(-\frac{r^2}{2\sigma_0^2}\right), \quad (6)$$

and adjust the width, σ_0 accordingly. We find $\sigma_0 = 0.23$ fm and 0.62 fm to satisfy the experimental value of N_{pp}^{1S}/N_{pp}^{2S} in pp , representing 2 different values of the

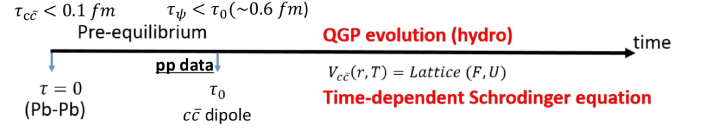


FIG. 1. Schematic diagram of a $c\bar{c}$ dipole evolution in heavy-ion collisions. After a hard collision in Pb-Pb collisions, the $c\bar{c}$ dipole is produced within $\tau_{c\bar{c}} < 0.1$ fm/c. The vacuum formation time of charmonium eigenstates, τ_ψ , is usually comparable to the QGP thermalization time τ_0 , but can become significantly longer at high momenta (Lorentz time dilation) and/or for color-screened potentials.

relative velocity between c and \bar{c} inside the dipole. The coefficient $A = 2/(\pi^{1/4}\sigma_0^{3/2})$ can be fixed to the normalization of the radial wavefunction, $\int r^2 dr \cdot |\phi_0(r)|^2 = 1$. We neglect medium effects in the HQ potential in pre-equilibrium stage prior to QGP formation, which amounts to conserving the initial fraction of eigenstates in the wavepackage. Therefore, we start the $c\bar{c}$ evolution from τ_0 where QGP reaches local equilibrium, cf. Fig. 1.

III. CHARMONIUM EVOLUTION IN STATIC MEDIUM

We start with a stationary $c\bar{c}$ in a static QGP medium at $T=1.5T_c$, initialized as a pure J/ψ state, cf. fig. 2. With the HQ potential as the free energy, $F(r, T)$, the J/ψ is a loosely bound dipole under these conditions. Its wavefunction expands outside, decreasing the overlap between the $c\bar{c}$ and the J/ψ eigen-function, while increasing the overlap with the $\psi(2S)$ eigen-function for times $t < 2$ fm/c. Subsequently, the $c\bar{c}$ dipole continues to expand decreasing the overlap with both J/ψ and $\psi(2S)$ components, although the latter stays larger than the former.

Next, we initialize the $c\bar{c}$ dipole wavefunction $\phi_0(r)$ at τ_0 with the ratio of direct J/ψ and $\psi(2S)$ yields in pp collisions using Eq. (6) with $\sigma_0 = 0.23$ fm, and evolve the $c\bar{c}$ dipole in the static medium with $T = 1.5T_c$. For the HQ potential we consider both the free energy, $F(r, T)$, and internal energy, $U(r, T)$. Similar to the previous case, the $\psi(2S)$ fraction increases at first and then decreases with time, cf. upper panel of Fig. 3. The pertinent double ratio of direct J/ψ and $\psi(2S)$ nuclear modification factors (without electromagnetic decay feeddowns from excited states), R_{AA}^{2S}/R_{AA}^{1S} , markedly increases with time, see lower panel of Fig. 3. This effect is significantly less pronounced when using the internal energy as HQ potential, since the much stronger attraction tends to keep the $c\bar{c}$ wavepackage more compact thus increasing (reducing) the overlap with the ground (excited) state, see the red line (with circles) in the lower panel of Fig. 3. A similar effect occurs at lower temperatures, e.g., at $T=T_c$. Even

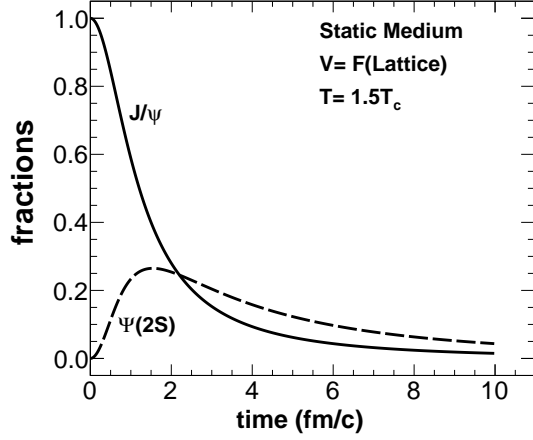


FIG. 2. The evolution of a stationary $c\bar{c}$ dipole in static medium with temperature $T = 1.5T_c$ and the free energy as the in-medium HQ potential. The initial wavefunction of the $c\bar{c}$ dipole is initialized as the J/ψ eigenstate. Solid and dashed lines are the fractions of J/ψ and $\psi(2S)$ in the $c\bar{c}$ wavefunction, respectively.

for the free energy, a large overall reduction in the time dependence of the double ratio is found, dropping below one for times $t > 2$ fm/c, cf. the line with triangles in the lower panel of Fig. 3.

We repeat the same calculations as described in the previous paragraph for larger width of the initial Gaussian wavepackage, $\sigma_0 = 0.62$ fm (which also recovers the empirical ψ' -over- J/ψ ratio in pp collisions), cf. Fig. 4. The enhancement in the ψ' R_{AA} is significantly delayed, as is the suppression of the J/ψ due to a smaller relative velocity of c and \bar{c} . These delays also manifest themselves in the slower growth of the R_{AA}^{2S}/R_{AA}^{1S} double ratio, starting out at values below one for all 3 scenarios considered, while still reaching well above one at later times for $T=1.5T_c$ and both potential types (less for stronger attraction). However, for temperatures near T_c , the double ratio drops dramatically within the first 1 fm/c of the evolution and stays small thereafter.

The inclusion of inelastic charmonium break-up reactions will introduce an imaginary part into our calculation which will lead to a suppression of both J/ψ and $\psi(2S)$, which is most likely stronger for the excited states. Therefore, we expect the fractions in the lower panels of Figs. 3 and 4 to be generally reduced. However, the transition mechanisms, $J/\psi \leftrightarrow \psi(2S)$, still exist within the $c\bar{c}$ dipole and may well play an important role in the finally observed R_{AA}^{2S}/R_{AA}^{1S} , including a nontrivial momentum dependence.

We have also employed this approach to bottomonia. Due to the large binding energy of $\Upsilon(1S)$, the screening of the HQ potential at the relevant distances, $r \sim \langle r \rangle_{\Upsilon(1S)}$, is less pronounced causing fewer transitions to occur. However, for the excited $\Upsilon(2S)$ and $\Upsilon(3S)$ states, we

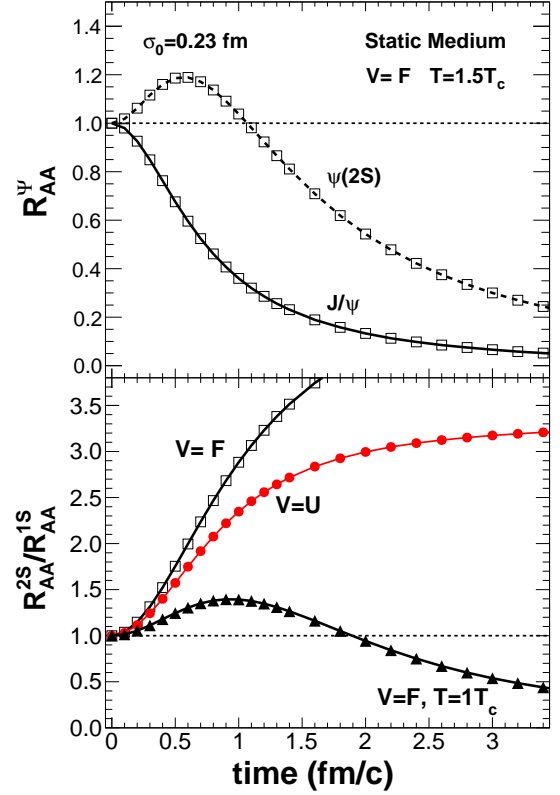


FIG. 3. (Color online) Upper panel: Direct J/ψ and $\psi(2S)$ nuclear modification factors, R_{AA} , as a function of time in a static medium with constant temperature, $T=1.5T_c$, and $V=F$. The initial $c\bar{c}$ wavefunction at τ_0 is taken as a Gaussian function of width $\sigma_0=0.23$ fm. Lower panel: double ratio, R_{AA}^{2S}/R_{AA}^{1S} , of direct ψ' over J/ψ corresponding to the curves in the upper panel (squares), and for $V=U$ (circles), as well as for a lower $T=T_c$ with $V=F$ (triangles).

expect transitions to be active, and thus provide an independent handle from the experimental side. This will be discussed in an upcoming manuscript.

IV. CONCLUSION

In summary, we have employed the time-dependent Schrödinger equation to simulate the internal evolution of HQ dipoles in a hot medium. An in-medium potential causes primordially produced correlated $c\bar{c}$ dipoles to undergo transitions between different asymptotic (vacuum) eigenstates over timescales which are significantly longer than the vacuum formation times of these states. In particular, for weak binding the dipole expands rather easily shifting wavefunction overlap from the ground to excited states. However, we have also found that whether these mechanisms result in an enhancement or suppression of the final double ratio, R_{AA}^{2S}/R_{AA}^{1S} , of

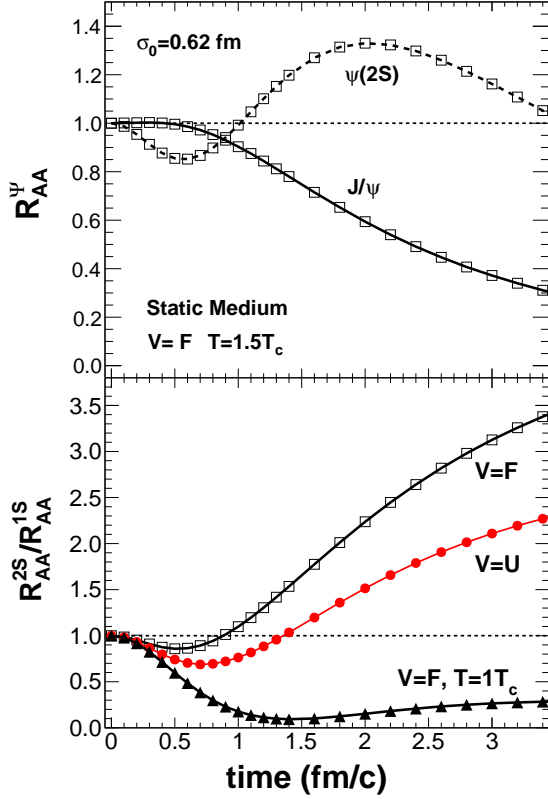


FIG. 4. (Color online) Same as Fig. 3, but for a larger Gaussian width of $\sigma_0=0.62$ fm for the initial $c\bar{c}$ wavepackage at τ_0 .

the charmonium nuclear modification factors, rather sensitively depends on the strength of the binding potential. A stronger attraction, i.e., less screening, due to either a stronger potential type (e.g., U rather

than F) or due to lower temperatures (e.g., near T_c), generally leads to smaller double ratios. Related effects are expected to occur in the $\Upsilon(2S)$ - $\Upsilon(3S)$ sector.

Acknowledgment. We thank P. Zhuang and Y. Liu for valuable discussions and input. This work has been supported by the US NSF under grant PHY-1614484, the NSFC and MOST grant Nos. 11335005, 11575093, 2013CB922000, 2014CB845400 and 11547043.

-
- [1] T. Matsui and H. Satz, Phys. Lett. B **178**, 416 (1986)
 - [2] H. Satz, J. Phys. G **32**, R25 (2006)
 - [3] B. Chen, Y. Liu, K. Zhou and P. Zhuang, Phys. Lett. B **726**, 725 (2013)
 - [4] V. Khachatryan *et al.* [CMS Collaboration], Phys. Rev. Lett. **113**, 262301 (2014)
 - [5] X. Du and R. Rapp, Nucl. Phys. A **943**, 147 (2015)
 - [6] B. Z. Kopeliovich *et al.*, Phys. Rev. D **92**, 034023 (2015)
 - [7] R. Katz and P. B. Gossiaux, Annals Phys. **368**, 267 (2016)
 - [8] L. Gerland *et al.*, Phys. Rev. C **69**, 014904 (2004)
 - [9] X. Zhao and R. Rapp, Proc. of 4th Winter Workshop on Nuclear Dynamics (WWND 2008); arXiv:0806.1239 [nucl-th].
 - [10] T. Song, C.M. Ko and S.H. Lee, Phys. Rev. C **91**, 044909 (2015)
 - [11] X. Zhao and R. Rapp, Phys. Lett. B **664**, 253 (2008)
 - [12] X. l. Zhu, P. f. Zhuang and N. Xu, Phys. Lett. B **607**, 107 (2005)
 - [13] M. Strickland and D. Bazow, Nucl. Phys. A **879**, 25 (2012)
 - [14] E. G. Ferreira, Phys. Lett. B **749**, 98 (2015)
 - [15] B. Chen, Phys. Rev. C **93**, 054905 (2016)
 - [16] H. Sorge, E. V. Shuryak and I. Zahed, Phys. Rev. Lett. **79**, 2775 (1997)
 - [17] S. Digal *et al.*, Eur. Phys. J. C **43**, 71 (2005)
 - [18] J. Crank and P. Nicolson, Adv. Comput. Math. **6**, 207 (1996)